INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, Second Semester, 2018-19
Statistics - IV, Backpaper Examination

1. Consider an $I \times J$ contingency table where the (i, j)th cell has count n_{ij} and probability p_{ij}. Find the maximum likelihood estimate of (p_{ij})
(a) when no restrictions are placed on the row and column factors;
(b) when it is known that the row and column factors are independent. [10]

2. Consider a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution with c.d.f. F and suppose we want to test $H_0 : F = F_0$ where F_0 is a fully specified c.d.f. Define the directional and non-directional Kolmogorov-Smirnov test statistics, D_n^+, D_n^- and D_n for testing H_0. Show that, under H_0,
(a) each of these statistics is distribution free;
(b) each of them converges to 0 in probability as $n \longrightarrow \infty$. [10]

3. Two methods, A and B, were used in a determination of the latent heat of fusion of ice. The investigators wished to check whether the methods differed, and if so, whether method B typically gave a higher reading. The following table gives the change in total heat from ice at $-.72°$C to $0°$C.

Method A	79.97	80.01	79.95	80.02	79.94
Method B	80.05	79.98	80.04	80.03	

Use an appropriate nonparametric method for this investigation. [10]

4. Consider the two-person, zero-sum game with the following loss matrix:

	a_1	a_2	a_3	a_4
θ_1	3	1	0	2
θ_2	0	2.5	3	1

Solve this game. [10]

5. Let $X_1, X_2, \ldots, X_{25}$ be a random sample from a normal population with mean θ and variance 25, where $-\infty < \theta < \infty$. Consider testing $H_0 : \theta = 10$ versus $H_1 : \theta = 12$. Suppose Type I error incurs twice as much loss (a positive quantity) as Type II error does, whereas there is no loss for correct decision. Show how to find the minimax test. [10]